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STUDYING THE TEMPERATURE FIELD IN THE RECORDING AND REPRODUCTION OF INFORMATION BY MEANS OF FOCUSED RADIATION

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UDC 536.12:539.377

The functions of an instantaneous spot source of heat acting on the boundary of layer separation have been constructed for a two-layer plate. The temperature field generated by a moving normally distributed source of radiation is studied in the recording and reproduction of information.

The most important component in the development of optical disk recording devices, as well as in the reproduction and storage of information is the study of the process involved in the propagation of heat generated in an active layer applied to a substrate transparent to optical radiation and focused with brief pulsed radiation (the thickness of the substrate considerable exceeds the thickness of the active layer). Under real conditions, since the three-dimensional distribution of radiation intensity is described by a complex law [1], it is a good idea to make it as simple as possible. In this connection, of practical interest is an examination of the problem pertaining to the heating of component parts in three-dimensional formulation from the standpoint of the heat sources which are effective at the point at which the layers are joined.

The solution of these problems for a two-layer plate can be found by means of the functions $G(r, r_0, \varphi, \varphi_0, z, \tau)$, satisfying the following equation, with discontinuous and singular coefficients:

$$\Delta G + \left(1 - \frac{\lambda_1}{\lambda_2}\right)\frac{\partial G}{\partial z}\bigg|_{z=z_1-0} \delta(z-z_1) - \left[\frac{1}{a_1} + \left(\frac{1}{a_2} - \frac{1}{a_1}\right) S(z-z_1)\right]\frac{\partial G}{\partial \tau} + \frac{1}{\lambda_2}\,\frac{\delta(r-r_0)}{r_0}\,\delta(\varphi-\varphi_0)\,\delta(z-z_1)\,\delta(\tau) = 0 \quad (1)$$

Institute of Applied Problems in Mathematics and Mechanics, Academy of Sciences of the Ukrainian SSR, L'vov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 57, No. 6, pp. 983-990, December, 1989. Original article submitted July 19, 1988.

 0022-0841/89/5701-1492$12.50 © 1990 Plenum Publishing Corporation

and the boundary conditions

$$\left.\frac{\partial G}{\partial z}\right|_{z=0}=0,\quad \left.G\right|_{z=z_2}=0,\quad \left.G\right|_{r\to\infty}=0,\quad \left.G\right|_{\tau=0}=0, \tag{2}$$

where

$$\Delta=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}+\frac{\partial^2}{\partial z^2}.$$

Let us note that Eq. (1) is equivalent to the heat-conduction equations $\Delta G_j = G_j/a_j$ ($j = 1, 2$) for each layer, as well as to the conditions of contact $G_1 = G_2$, $\lambda_2(\partial G_2/\partial z) - \lambda_1(\partial G_1/\partial z) = -[\delta(r - r_0)/r_0]\delta(\varphi - \varphi_0)\delta(z - z_1)\delta(\tau)$ for $z = z_1$, $G = G_1 + (G_2 - G_1)S(z - z_1)$.

Applying the Fourier, Hankel, and Laplace transforms with respect to the variables φ, r, τ to (1) and (2), respectively, we obtain the following ordinary differential equation

$$\frac{d^2\bar{G}}{dz^2}-[\varepsilon_1^2+(\varepsilon_2^2-\varepsilon_1^2)S(z-z_1)]\bar{G}- \tag{3}$$

$$-\left(\frac{\lambda_1}{\lambda_2}-1\right)\left.\frac{d\bar{G}}{dz}\right|_{z=z_1-0}\delta(z-z_1)=-\frac{1}{\lambda_2}A(\eta,\ \nu)\delta(z-z_1)$$

and the boundary conditions

$$\left.\frac{d\bar{G}}{dz}\right|_{z=0}=0,\quad \left.\bar{G}\right|_{z=z_2}=0. \tag{4}$$

Here

$$A(\eta,\ \nu)=J_\nu(\eta r_0)\begin{cases}\cos\nu\varphi_0 \text{ with } \nu=0,\ 2,\ 4,\ \ldots,\\ \sin\nu\varphi_0 \text{ with } \nu=1,\ 3,\ 5,\ \ldots,\end{cases}$$

$$\varepsilon_j=\sqrt{\frac{s}{a_j}+\eta^2}\ (j=1,\ 2).$$

Following [2], we represent the solution of problem (3), (4) in the form

$$\bar{G}=\frac{A(\eta,\ \nu)}{Q\varepsilon_2\lambda_2}\Big\{\operatorname{sh}\varepsilon_2(z_2-z_1)\Big[\operatorname{ch}\varepsilon_1 z+(\operatorname{ch}\varepsilon_1 z_1\operatorname{ch}\varepsilon_2(z-z_1)+\frac{\varepsilon_1}{\varepsilon_2}\frac{\lambda_1}{\lambda_2}\times \tag{5}$$

$$\times\operatorname{sh}\varepsilon_1 z_1\operatorname{sh}\varepsilon_2(z-z_1)-\operatorname{ch}\varepsilon_1 z)S(z-z_1)\Big]-Q\operatorname{sh}\varepsilon_2(z-z_1)S(z-z_1)\Big\},$$

$$Q=\operatorname{ch}\varepsilon_1 z_1\operatorname{ch}\varepsilon_2(z_2-z_1)+\frac{\varepsilon_1}{\varepsilon_2}\frac{\lambda_1}{\lambda_2}\operatorname{sh}\varepsilon_1 z_1\operatorname{sh}\varepsilon_2(z_2-z_1).$$

Turning in (5) from the images to originals, using the theory of expansion for the integral Laplace transform, we will write the solution of problem (1), (2) in the form

$$G=\frac{1}{\pi}\frac{a_1}{\lambda_2}\int_0^\infty \eta J_0(\eta d_0)\sum_{n=1}^{\infty}\frac{\psi(\eta,\ z,\ \mu_n)}{D(\eta,\ \mu_n)}\exp(-\mu_n^2\tau)\,d\eta, \tag{6}$$

where

$$d_0^2=r^2+r_0^2-2rr_0\cos(\varphi-\varphi_0);$$

$$\psi(\eta,\ z,\ \mu)=\frac{\delta_1}{\delta_2}\sin\delta_2(z_2-z_1)\Big[\cos\delta_1 z+(\cos\delta_1 z_1\cos\delta_2(z-z_1)-$$

$$-\frac{\delta_1}{\delta_2}\frac{\lambda_1}{\lambda_2}\sin\delta_1 z_1\sin\delta_2(z-z_1)-\cos\delta_1 z)S(z-z_1)\Big];$$

$$D(\eta,\ \mu)=\Lambda_1\sin\delta_2(z_2-z_1)\sin\delta_1 z_1+\Lambda_2\sin\delta_2(z_2-z_1)\cos\delta_1 z_1+$$

$$+\Lambda_3\cos\delta_2(z_2-z_1)\sin\delta_1 z_1;$$

$$\Lambda_1=\frac{1}{\delta_2}\frac{\lambda_1}{\lambda_2}\left(1-\frac{a_1}{a_2}\frac{\delta_1^2}{\delta_2^2}\right),\ \Lambda_2=\frac{\delta_1}{\delta_2}\left(\frac{\lambda_1}{\lambda_2}z_1+\frac{a_1}{a_2}(z_2-z_1)\right);$$

$$\Lambda_3 = z_1 + \frac{\lambda_1}{\lambda_2}\frac{\delta_1^2}{\delta_2^2}\frac{a_1}{a_2}(z_2 - z_1),\ \delta_j = \sqrt{\frac{\mu^2}{a_j} - \eta^2}\ (j = 1, 2);$$

μ_n are the roots of the equation $\cos\delta_1 z_1 \cos\delta_2(z_2 - z_1) - (\delta_1/\delta_2)(\lambda_1/\lambda_2)\sin\delta_1 z_1 \sin\delta_2(z_2 - z_1) = 0$.

A solution in the form (6) can be used effectively for numerical calculations, given sufficiently large values for the time. Let us dwell in some detail on obtaining a solution for boundary-value problem (1), (2), such as is convenient from the standpoint of realization with small time values.

With large values of s for $a_2 < a_1$, the expression for $\bar{G}$ can be transformed as follows:

$$\bar{G} = \frac{A(\eta, \nu)}{\lambda_2 \varepsilon_1}\frac{1}{Q_2}[\exp(-\varepsilon_1(z_1 - z))(1 + \exp(-2\varepsilon_1 z))S(z_1 - z) + \exp(-\varepsilon_2(z - z_1))(1 + \exp(-2\varepsilon_1 z_1))S(z - z_1)]\sum_{n=0}^{\infty} u^n, \quad (7)$$

where

$$u = \frac{1}{Q_2}\left(\sqrt{\frac{a_1}{a_2}} - \frac{\varepsilon_2}{\varepsilon_1}\right)(1 + \exp(-2\varepsilon_1 z_1));$$

$$Q_2 = \sqrt{\frac{a_1}{a_2}} + \frac{\lambda_1}{\lambda_2} + \left(\sqrt{\frac{a_1}{a_2}} - \frac{\lambda_1}{\lambda_2}\right)\exp(-2\varepsilon_1 z_1),$$

where $|u| < 1$.

Since

$$\frac{\varepsilon_2}{\varepsilon_1} = \sqrt{\frac{a_1}{a_2}}\sqrt{1 + \frac{\eta^2 a_1}{s + \eta^2 a_1}\left(\frac{a_2}{a_1} - 1\right)} \quad (8)$$

and $a_2 < a_1$ for $\varepsilon_2/\varepsilon_1$ the following expansion is valid:

$$\frac{\varepsilon_2}{\varepsilon_1} = \sqrt{\frac{a_1}{a_2}}\left(1 + \frac{1}{2}\frac{\eta^2 a_1}{s + \eta^2 a_1}\left(\frac{a_2}{a_1} - 1\right) - \frac{1}{8}\left(\frac{a_2}{a_1} - 1\right)^2\left(\frac{\eta^2 a_1}{s + \eta^2 a_1}\right)^2 + \ldots\right). \quad (9)$$

Limiting ourselves in (9) to two terms, i.e., assuming that

$$\frac{\varepsilon_2}{\varepsilon_1} \approx \sqrt{\frac{a_1}{a_2}}\left(1 + \frac{1}{2}\frac{\eta^2 a_1}{s + \eta^2 a_1}\left(\frac{a_2}{a_1} - 1\right)\right) \quad (10)$$

and substituting (10) into (7), keeping in mind in this case only the first two terms of the series in (7), since the latter contains terms of a higher order of smallness, and utilizing the representations

$$\frac{1}{Q_2} = \frac{1}{B_2}\sum_{p=0}^{\infty}\left(\frac{B_1}{B_2}\right)^p \exp(-2\varepsilon_1 z_1 p),$$

$$\frac{1}{Q_2^2} = \frac{1}{B_2^2}\sum_{p=1}^{\infty} p\left(\frac{B_1}{B_2}\right)^{p-1}\exp(-2\varepsilon_1 z_1(p - 1)),$$

we obtain

$$G \approx \frac{A(\eta, \nu)}{\lambda_2 \varepsilon_1}\left[\frac{1}{B_2}\sum_{p=0}^{\infty}\left(\frac{B_1}{B_2}\right)^p \exp(-2\varepsilon_1 z_1 p) + \frac{1}{2}\frac{1}{B_2^2}\sqrt{\frac{a_1}{a_2}}\left(1 - \frac{a_2}{a_1}\right)\frac{\eta^2 a_1}{s + \eta^2 a_1}\sum_{p=1}^{\infty} p\left(\frac{B_1}{B_2}\right)^{p-1} \times \exp(-2\varepsilon_1 z_1(p - 1))\right][\exp(-\varepsilon_1(z_1 - z))(1 + \exp(-2\varepsilon_1 z))S(z_1 - z) + \exp(-\varepsilon_2(z - z_1))(1 + \exp(-2\varepsilon_1 z_1))S(z - z_1)], \quad (11)$$

where

$$B_1=\frac{\lambda_1}{\lambda_2}-\sqrt{\frac{a_1}{a_2}};\ B_2=\frac{\lambda_1}{\lambda_2}+\sqrt{\frac{a_1}{a_2}}.$$

Making the transition in (11) from images to the originals, using the theorem of displacement and convolution of the Laplace transform, as well as the inversion formulas for the Hankel and Fourier transforms, we have

$$G\approx\Bigg[\frac{1}{4}\frac{(\pi\tau)^{-\frac{3}{2}}}{B_2\lambda_2\sqrt{a_1}}\exp\left(-\frac{d_0^2}{4a_1\tau}\right)\sum_{l=1}^{2}\sum_{p=0}^{\infty}\left(\frac{B_1}{B_2}\right)^p\exp\left(-\frac{(2z_1p+\beta_{l1})^2}{4a_1\tau}\right)+ \tag{12}$$

$$+\frac{1}{8}(\pi\lambda_2\sqrt{a_2})^{-1}\left(1-\frac{a_2}{a_1}\right)(B_2\tau)^{-2}\exp\left(-\frac{d_0^2}{4a_1\tau}\right)\times$$

$$\times\left(1-\frac{d_0^2}{4a_1\tau}\right)F_1(\tau,\ z)\Bigg]S(z_1-z)+\Bigg[\frac{\beta_{12}}{8\pi^2}(B_2\lambda_2)^{-1}\times$$

$$\times\int_0^\tau\frac{\exp(-d_1)}{g\sqrt{u}(\tau-u)^{\frac{3}{2}}}\sum_{l=1}^{2}\sum_{p=0}^{\infty}\left(\frac{B_1}{B_2}\right)^p\exp\left(-\frac{\alpha_{lp}}{4a_1u}\right)du+$$

$$+\frac{\beta_{12}}{16\pi^{\frac{3}{2}}}\sqrt{\frac{a_1}{a_2}}(a_1-a_2)\frac{1}{B_2^2}\int_0^\tau\frac{\exp(-d_1)}{g^2(\tau-u)^{\frac{3}{2}}}\times\left(1-\frac{d_0^2}{4g}\right)R_1(u)\,du\Bigg]S(z-z_1).$$

Here

$$\beta_{1j}=(-1)^j\sqrt{\frac{a_1}{a_j}}(z-z_1),\ \beta_{2j}=2z_1+\sqrt{\frac{a_1}{a_j}}(z-z_1)\ (j=1,\ 2),$$

$$\beta_{31}=3z_1-z,\ \beta_{41}=3z_1+z,\ F_1(\tau,\ z)=$$

$$=2\sqrt{\tau}\sum_{l=1}^{4}\sum_{p=1}^{\infty}p\left(\frac{B_1}{B_2}\right)^{p-1}\operatorname{ierfc}\left(\frac{\alpha_{3p}+\beta_{l1}}{2\sqrt{a_1\tau}}\right),$$

$$R_1(u)=2\sqrt{u}\sum_{l=1}^{4}\sum_{p=1}^{\infty}p\left(\frac{B_1}{B_2}\right)^{p-1}\operatorname{ierfc}\left(\frac{\alpha_{lp}}{2\sqrt{a_1u}}\right),$$

$$g=a_2\tau+(a_1-a_2)u,\ d_1=\frac{d_0^2}{4g}+\frac{(z-z_1)^2}{4a_1(\tau-u)},$$

$$\alpha_{1p}=2pz_1,\ \alpha_{2p}=2(p+1)z_1,\ \alpha_{3p}=2(p-1)z_1,\ \alpha_{4p}=\alpha_{1p}.$$

If we limit ourselves in (9) only to the first term, i.e., if we assume $\varepsilon_2/\varepsilon_1\approx\sqrt{a_1/a_2}$, then we obtain the following expression for G_j ($j = 1, 2$):

$$G_j\approx\frac{1}{4}\frac{(\pi\tau)^{-\frac{3}{2}}}{B_2\lambda_2\sqrt{a_1}}\exp\left(-\frac{d_0^2}{4a_1\tau}\right)\sum_{l=1}^{2}\sum_{p=0}^{\infty}\left(\frac{B_1}{B_2}\right)^p\exp\left(-\frac{(2z_1p+\beta_{lj})^2}{4a_1\tau}\right). \tag{13}$$

If in the expressions for ε_1, ε_2 in (7), following [3], we assume that $\eta = 0$, then

$$G_j\approx\frac{\sqrt{a_1}}{\lambda_2B_2}\frac{1}{\sqrt{\pi\tau}}\frac{\delta(r-r_0)}{r_0}\delta(\varphi-\varphi_0)\sum_{l=1}^{2}\sum_{p=0}^{\infty}\left(\frac{B_1}{B_2}\right)^p\exp\left(-\frac{(2z_1p+\beta_{lj})^2}{4a_1\tau}\right). \tag{14}$$

Formulas (14), (13), and (12) are successive approximations for the determination of the functions of an instantaneous spot source of heat. Let us take note of the fact that from (13) we can obtain a precise expression for the function G for a uniform half-space:

$$G=\frac{1}{8}\frac{(\pi\tau)^{-\frac{3}{2}}}{\lambda\sqrt{a}}\exp\left(-\frac{d_0^2}{4a\tau}\right)\sum_{l=1}^{2}\exp\left(-\frac{\beta_{l1}^2}{4a\tau}\right).$$

On the basis of the constructed functions $G(r,\ r_0,\ \varphi,\ \varphi_0,\ z,\ \tau)$ we will write out the expression of the temperature field in the two-layer plate, said field generated by the action of heat sources of density $W(r,\varphi,\tau)\delta(z-z_1)$, where $W(r,\varphi,\tau)$ is some arbitrary function of the coordinates r, φ and of time τ:

$$t(r,\ \varphi,\ z,\ \tau)=\int_0^{\tau}\int_0^{\infty}\int_0^{2\pi} r_0 W(r_0,\ \varphi_0,\ v)\, G(r,\ r_0,\ \varphi,\ \varphi_0,\ z,\ \tau-v)\, d\varphi_0 dr_0 dv.$$

In the case of a normally distributed pulse-radiation heat source moving about a circle at a constant angular velocity ω, the expression for the temperature in the active layer assumes the form

$$t_1(r,\ \varphi,\ z,\ \tau)=\int_0^{\tau}\int_0^{\infty}\int_0^{2\pi} r_0 q(v)\exp\{-k[r_0^2+c^2-2r_0 c\cos(\varphi_0-\omega v)]\}\, G_1(r,\ r_0,\ \varphi,\ \varphi_0,\ z,\ \tau-v)\, d\varphi_0 dr_0 dv, \tag{15}$$

$$q(v)=q_0\sum_{n=0}^{m-1}[S_+(\tau-b_n)-S_+(\tau-b_n-\tau_1)],$$

$b_n = n(\tau_1 + \tau_2)$, and c is the distance from the coordinate origin to the center of the spot. Substituting (12) into (15) and carrying out the integration, we write

$$t_1(r,\ \varphi,\ z,\ \tau)=t_{11}(r,\ \varphi,\ z,\ \tau)+t_{12}(r,\ \varphi,\ z,\ \tau), \tag{16}$$

where

$$t_{11}(r,\ \varphi,\ z,\ \tau)=\frac{\sqrt{a_1}}{B_2}\frac{q_0}{\lambda_2\sqrt{\pi}}\sum_{n=0}^{m-1}\Big[S_+(\tau-b_n)\int_0^{\tau-b_n}\Theta_1(r,\ \varphi,\ z,\ u)\,du - S_+(\tau-b_n-\tau_1)\int_0^{\tau-b_n-\tau_1}\Theta_1(r,\ \varphi,\ z,\ u)\,du\Big], \tag{17}$$

$$t_{12}(r,\ \varphi,\ z,\ \tau)=\frac{1}{2B_2^2}\frac{q_0}{\lambda_2\sqrt{a_2}}(a_1-a_2)\sum_{n=0}^{m-1}\Big[S_+(\tau-b_n)\times \int_0^{\tau-b_n}\Theta_2(r,\ \varphi,\ z,\ u)\,du - S_+(\tau-b_n-\tau_1)\int_0^{\tau-b_n-\tau_1}\Theta_2(r,\ \varphi,\ z,\ u)\,du\Big], \tag{18}$$

$$\Theta_1(r,\ \varphi,\ z,\ u)=\frac{\exp\left(-\dfrac{kd^2}{u_1}\right)}{u_1\sqrt{u}}\sum_{l=1}^{2}\sum_{p=0}^{\infty}\left(\frac{B_1}{B_2}\right)^p\exp\left(-\frac{(2z_1 p+\beta_{l1})^2}{4a_1 u}\right),$$

$$\Theta_2(r,\ \varphi,\ z,\ u)=\exp\left(-\frac{kd^2}{u_1}\right)\left(\frac{u_1-kd^{2-1}}{u_1^2 u}+\frac{kd^2}{u_1^3 u}\right)F_1(u,\ z),$$

$$d^2=r^2+c^2-2rc\cos(\varphi-\omega(\tau-u)),\ u_1=1+4ka_1 u.$$

Let us examine certain special cases in the solution of the heat-conduction problem. If we use (13) with j = 1, in order to determine the temperature field in the active layer, we find that $t_1(r,\varphi,z,\tau)\approx t_{11}(r,\varphi,z,\tau)$. Substituting expression (14) into (15) with j = 1, we find that

$$t_1(r,\ \varphi,\ z,\ \tau)\approx\frac{\sqrt{a_1}}{B_2}\frac{q_0}{\lambda_2\sqrt{\pi}}\sum_{n=0}^{m-1}\Big[S_+(\tau-b_n)\int_0^{\tau-b_n}\Theta_3(r,\ \varphi,\ z,\ u)\,du - S_+(\tau-b_n-\tau_1)\int_0^{\tau-b_n-\tau_1}\Theta_3(r,\ \varphi,\ z,\ u)\,du\Big],$$

where

$$\Theta_3(r,\ \varphi,\ z,\ u)=\frac{\exp(-kd^2)}{\sqrt{u}}\sum_{l=1}^{2}\sum_{p=0}^{\infty}\exp\left(-\frac{(2z_1 p+\beta_{l1})^2}{4a_1 u}\right).$$

With $4ka_1\tau \ll 1$ the same result follows out of (16).

For a uniform half-space the exact solution of the heat-conduction problem is presented in the form

$$t(r,\ \varphi,\ z,\ \tau)=\frac{\sqrt{a}}{2\lambda}\frac{q_0}{\sqrt{\pi}}\sum_{n=0}^{m-1}\Big[S_+(\tau-b_n)\int_0^{\tau-b_n}\Theta(r,\ \varphi,\ z,\ u)\,du - S_+(\tau-b_n-\tau_1)\int_0^{\tau-b_n-\tau_1}\Theta(r,\ \varphi,\ z,\ u)\,du\Big],$$

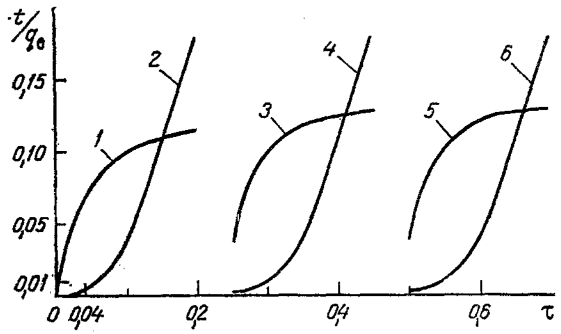

Fig. 1

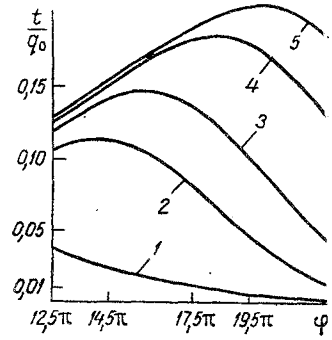

Fig. 2

Fig. 1. The relationship between $t/q_0(m^2 \cdot K \cdot W^{-1})$ and $\tau(10^{-6}$ sec) for various values of φ: 1) $\varphi = 0$; 2) $\varphi = \omega\tau_1$; 3) $\varphi = \omega(\tau_1 + \tau_2)$; 4) $\varphi = \omega(2\tau_1 + \tau_2)$; 5) $\varphi = 2\omega(\tau_1 + \tau_2)$; 6) $\varphi = \omega(3\tau_1 + 2\tau_2)$.

Fig. 2. The relationship betwwen $t/q_0(m^2 \cdot K \cdot W^{-1})$ and φ (rad) at various instants of time τ: 1) $\tau = 0.25 \cdot 10^{-6}$ sec; 2) $\tau = 0.31 \cdot 10^{-6}$; 3) $\tau = 0.35 \cdot 10^{-6}$; 4) $\tau = 0.41 \cdot 10^{-6}$; 5) $\tau = 0.45 \cdot 10^{-6}$ sec.

where

$$\Theta(r,\ \varphi,\ z,\ u) = \frac{\exp\left(-\dfrac{kd^2}{1+4kau}\right)}{(1+4kau)\sqrt{u}} \sum_{l=1}^{2} \exp\left(-\frac{\beta_{l1}^2}{4au}\right).$$

In the case of a nonmoving heat source

$$t_1(r',\ z,\ \tau) = t_{11}(r',\ z,\ \tau) + t_{12}(r',\ z,\ \tau),$$

where

$$t_{11}(r',\ z,\ \tau) = \frac{\sqrt{a_1}}{B_2}\,\frac{q_0}{\lambda_2\sqrt{\pi}} \sum_{n=0}^{m-1}\left[S_+(\tau - b_n)\int_0^{\tau-b_n}\Theta_1(r',\ z,\ u)\,du - \right.$$

$$\left. - S_+(\tau - b_n - \tau_1)\int_0^{\tau-b_n-\tau_1}\Theta_1(r',\ z,\ u)\,du\right];$$

$$t_{12}(r',\ z,\ \tau) = \frac{1}{2B_2^2}\,\frac{q_0}{\lambda_2\sqrt{a_2}}(a_1 - a_2)\sum_{n=0}^{m-1}\left[S_+(\tau - b_n)\int_0^{\tau-b_n}\Theta_2(r',\ z,\ u)\,du - \right.$$

$$\left. - S_+(\tau - b_n - \tau_1)\int_0^{\tau-b_n-\tau_1}\Theta_2(r',\ z,\ u)\,du\right];$$

$$\Theta_1(r',\ z,\ u) = \frac{\exp\left(-\dfrac{kr'^2}{u_1}\right)}{u_1\sqrt{u}}\sum_{l=1}^{2}\sum_{p=0}^{\infty}\left(\frac{B_1}{B_2}\right)^p \exp\left(-\frac{(2z_1p + \beta_{l1})^2}{4a_1u}\right);$$

$$\Theta_2(r',\ z,\ u) = \exp\left(-\frac{kr'^2}{u_1}\right)\left(\frac{u_1 - kr'^2 - 1}{u_1^2u} + \frac{kr'^2}{u_1^3u}\right)F_1(u,\ z),$$

r' is reckoned from the center of heating spot. Where $4k\,a_1\tau \ll 1$, to determine $t_1(r',\ z,\ \tau)$, we have the simple formula:

$$t_1(r',\ z,\ \tau) = \frac{2q_0}{\lambda_2}\,\frac{\sqrt{a_1}}{B_2}\sum_{n=0}^{m-1}\sum_{l=1}^{2}\sum_{p=0}^{\infty}\left[S_+(\tau - b_n)\sqrt{\tau - b_n}\ \mathrm{ierfc}\,\frac{2z_1p + \beta_{l1}}{2\sqrt{a_1}\sqrt{\tau - b_n}} - \right.$$

$$\left. - S_+(\tau - b_n - \tau_1)\sqrt{\tau - b_n - \tau_1}\ \mathrm{ierfc}\,\frac{2z_1p + \beta_{l1}}{2\sqrt{a_1}\sqrt{\tau - b_n - \tau_1}}\right].$$

Based on (16) we calculated the temperature field at the point at which the layers were joined, with the following values for the parameters: $z_1 = 0.12 \cdot 10^{-6}$ m, $\tau_1 = 0.2 \cdot 10^{-6}$ sec, $\tau_2 = 0.5 \cdot 10^{-7}$ sec, $\omega = 50\pi$ rad/sec, $c = 0.045$ m, $\lambda_1 = 8.37$ W/(m·K), $\lambda_2 = 0.2$ W/(m·K), $a_1 = 0.3 \cdot 10^{-5}$ m²/sec, $a_2 = 0.7 \cdot 10^{-7}$ m²/sec, $k = 0.307787 \cdot 10^{13}$ m⁻², $r = c$. The results of these calculations can be found in Figs. 1 and 2.

We can see from these figures that the initial pulse has little effect on the change in temperature at those points through which the center of the heating spot passes during the second pulse, whereas the second pulse has virtually no influence on the change in temperature at those points through which the spot center passes during the third pulse. It thus follows from the foregoing that in order to study the process of heat propagation it is sufficient to limit ourselves to several pulses.

The maximum temperature is achieved at the end of the pulse at those points through which the heating spot center passes within $(0.14\text{-}0.16)\cdot 10^{-6}$ sec from the instant of pulse onset.

Let us note that for the calculations that we have carried out here the absolute magnitude of $t_{12}(r, \varphi, z_1, \tau)$ amounts to no more than 10% of the total $t_1(r, \varphi, z_1, \tau)$. Should it become necessary to refine the temperature values, we must take into consideration a larger number of terms in expansion (9) and in the series in (7).

NOTATION

t, temperature in the plate; t_1, temperature in active layer; r, φ, z, cylindrical coordinates; τ, time; z_2, thickness of two-layer plate; z_1, thickness of active layer; (r_0, φ_0, z_1), effective point of instantaneous heat source at initial instant of time; λ_1 and a_1, λ_2 and a_2 coefficients of thermal conductivity and thermal diffusivity for the active layer and the substrate, respectively; λ and a, coefficients of thermal conductivity and thermal diffusivity in the uniform half-space; k, concentration factor; q_0, surface density of heat-source power; τ_1, pulse duration; τ_2, pause duration; m, number of pulses and pauses; s, Laplace transform parameter; η, Hankel transform parameter; $\delta(x)$, Dirac δ function; S(z), Heaviside unit function; $S_+(x)$, unit asymmetric function; $J_\nu(x)$, Bessel function of the first kind of the ν-th order; ierfc(x), repeated integral of additional error function.

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